

Problem Set 6 – The Marginal Principle

*Estimated time: about an hour. Type your responses and submit a single PDF *ON CANVAS*. Show your work on anything you compute. Label all axes and key points on graphs. A photo of a hand-drawn sketch is fine.*

Part 1: Marginal Benefits and Marginal Costs

1. A high school junior is deciding how many full-length SAT practice tests to take. Her expected score depends on how many she takes:

Practice tests	Expected score
0	1000
1	1080
2	1140
3	1180
4	1200
5	1210

Each test takes a Saturday morning. She values those mornings at \$20, \$30, \$45, \$60, and \$80 for the 1st through 5th test. She values each point on the SAT at \$1.

- (a) Find the MB of each test in points and in dollars.
 - (b) How many tests should she take? Explain using MB and MC.
 - (c) Is MB diminishing, constant, or increasing? Give one reason why. Is MC diminishing, constant, or increasing? Give one reason why.
 - (d) Sketch MB and MC on one graph with tests on the horizontal axis. Mark her choice.
 - (e) A college she likes offers a scholarship that rises with SAT scores, so she now values each point at \$2. How many tests should she take now? On your graph, which curve shifts?
2. A high school has 1,200 seniors. Each counselor costs \$65,000. The number of seniors who complete the FAFSA depends on the number of counselors:

Counselors	Students per counselor	FAFSA completions
2	600	300
3	400	420
4	300	500
5	240	550
6	200	580

The district values each additional FAFSA completion at \$1,000.

- (a) Find the MB of the 3rd, 4th, 5th, and 6th counselor, in completions and in dollars.
- (b) How many counselors should the school have?
- (c) A professional association recommends no more than 250 students per counselor. How many counselors would that take? At what value per completion would it be worth meeting the recommendation?
- (d) With 4 counselors, the average counselor produces $500/4 = 125$ completions, or \$125,000, which is far more than the \$65,000 cost. A board member says this proves the school should hire more counselors. What is wrong with this argument?

Part 2: The Shape of Marginal Cost

- 3. A school has 132 students in 3rd grade and 5 teachers. Each teacher costs \$65,000. The state is considering capping 3rd grade classes at 22 students, and an advocacy group wants a cap of 18.
 - (a) What is the average class size now?
 - (b) How many teachers does the school need under a cap of 22? Under a cap of 18? What is the average class size under each?
 - (c) Find the cost per one-student drop in average class size going from today to the 22 cap, and from the 22 cap to the 18 cap. Is MC increasing, constant, or decreasing?
 - (d) Under the 22 cap, one more student enrolls, for 133 total. What is the MC of that student? Why is this a lumpy decision?
- 4. For each, say whether marginal cost is increasing, constant, or decreasing (or describe another pattern). Explain in one or two sentences.
 - (a) A university adds students to a lecture course in a 300-seat hall. 250 students are enrolled now.
 - (b) A district adds pre-K slots. Its elementary schools have some empty rooms, but once those fill it must lease new buildings.
 - (c) A university prints one more diploma.
 - (d) A full-time student adds hours to a part-time job each week. Measure the cost in what the student gives up.

Part 3: Applying the Idea

5. A superintendent has \$300,000. An instructional coach costs \$100,000. A tutoring program at one school costs \$50,000. MB is the number of additional students who reach proficiency:

	Coaches	Tutoring programs
1st	60	40
2nd	45	30
3rd	30	20
4th	15	15
5th	–	10
6th	–	5

- Find the MB per \$10,000 for each coach and each tutoring program.
 - How should the superintendent spend the \$300,000? How many students reach proficiency? Compare your answer to spending it all on coaches and to spending it all on tutoring.
 - The budget is cut to \$200,000. Now how should she spend it?
 - A board member says, “A coach helps 60 students and a tutoring program only helps 40. Put all the money into coaches.” Respond in two or three sentences.
6. A 17-year-old has finished 11th grade and is deciding whether to finish high school. Finishing means giving up a full-time job, which costs her \$10,000 in earnings and effort. She values the diploma at \$8,000 today. Society gains another \$6,000 from her higher taxes and lower use of public services.
- Does she finish? Explain using her own MB and MC.
 - Is her finishing good for society? Explain.
 - Why might her own MB be smaller than the value of the diploma over her whole life?
 - The state offers a cash bonus for graduating. What is the smallest bonus that gets her to finish?
 - Some states instead require students to stay in school until 18. Give one advantage and one drawback of this approach compared to the bonus.
7. True or False, and explain in one or two sentences.
- “If smaller classes raise achievement, a district should keep cutting class size as long as it can afford to.”

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- (b) "If marginal benefit is falling, total benefit must be falling too."
- (c) "The district already paid \$300,000 for a three-year software license, so it should keep using the software even though teachers say it doesn't help."
- (d) "A program with a large average effect in a study should be expanded statewide."