

## Problem Set 8 – Equilibrium and Peer Effects

*Estimated time: about an hour. Type your responses and submit a single PDF \*ON CANVAS\*. Show your work on anything you compute. Label all axes and key points on graphs. A photo of a hand-drawn sketch is fine.*

### Part 1: What Counts as a Peer Effect

1. For each finding below, say whether the most natural explanation is an **endogenous** effect (your outcome responds to your classmates' outcomes), a **contextual** effect (your outcome responds to your classmates' characteristics), or a **correlated** effect (you and your classmates look alike because you share something else). Then name one piece of evidence that would let you rule out the other two.
  - (a) Students in classrooms with more classmates who have been retained a grade score lower on year-end tests.
  - (b) A student's attendance rises in months when her close friends' attendance rises.
  - (c) Graduates of a selective charter school earn more at age 30 than graduates of the neighborhood school.
  - (d) Boys assigned to cohorts with a higher share of girls are suspended less often.
  - (e) Teachers whose colleagues have adopted a new curriculum are more likely to adopt it themselves.
  - (f) Every student in one elementary school posts score gains in the year a new principal arrives.
2. Ann and Ben are assigned as roommates by lottery. Each one's end-of-year score depends on her or his own ability and on the roommate's score:

$$\text{Ann's score} = 50 + 5 \times (\text{Ann's ability}) + 0.2 \times (\text{Ben's score})$$

$$\text{Ben's score} = 50 + 5 \times (\text{Ben's ability}) + 0.2 \times (\text{Ann's score})$$

Ann's ability is 4 and Ben's is 2.

- (a) Solve for both scores. (Substitute one equation into the other and solve for a single unknown, then back out the other score.) Verify your answer satisfies both equations.
- (b) Now suppose Ben's ability had been 4 rather than 2. Solve again. How much higher is Ann's score, and why does Ann gain at all when nothing about Ann changed?

- (c) A researcher observes both scores and wants to recover the 0.2. Explain in two or three sentences why regressing Ann's score on Ben's score will not give it, even though the roommate assignment was random.
- (d) Instead the researcher regresses Ann's score on Ben's *ability*, measured before the two ever met. Which of the three effects from Problem 1 does this estimate? Why is it recoverable when the other is not?
- (e) Random assignment removes the sorting problem. Name two things it does *not* remove.

## Part 2: Disruption and Class Size

3. A middle school's classes are disrupted from time to time, and no one learns while a disruption is underway. Let  $p$  be the chance that any one student is *not* disrupting at a given moment, so a class of  $n$  students is learning a share  $p^n$  of the time. The school values a year of undisrupted learning at  $V = \$20,000$  per student, and a teacher costs  $W = \$60,000$ . Spread over a class of  $n$ , the teacher costs  $W/n$  per student, so the school's net benefit per student is  $Vp^n - W/n$ .

| $n$ | $p^n$ at $p = 0.90$ | $p^n$ at $p = 0.98$ |
|-----|---------------------|---------------------|
| 5   | 0.590               | 0.904               |
| 10  | 0.349               | 0.817               |
| 15  | 0.206               | 0.739               |
| 20  | 0.122               | 0.668               |
| 25  | 0.072               | 0.603               |

- (a) Compute the teacher cost per student,  $W/n$ , at each class size.
- (b) For  $p = 0.90$ , compute  $Vp^n$  and the net benefit at each class size. What is the best class size?
- (c) Do the same for  $p = 0.98$ . What is the best class size there? State in one sentence what the comparison says about which students belong in larger classes.
- (d) Teacher pay rises to \$120,000. Redo the  $p = 0.98$  column. Which way does the best class size move, and what is the economic reason?
- (e) Using your two columns, compare how much learning time is lost going from a class of 5 to a class of 25 for each group. Where would you expect a class-size reduction to show the largest effect on test scores, and why?
- (f) The model says a teacher who raises  $p$  is valuable. Explain how a teacher might raise  $p$  without being any better at explaining the subject matter.

4. The same school enrolls 300 students: 150 with  $p = 0.90$  and 150 with  $p = 0.98$ . Teachers still cost \$60,000 and  $V$  is still \$20,000. You will need two more values:  $0.90^{12} = 0.282$  and  $0.98^{12} = 0.785$ .
- (a) **Policy A (group by  $p$ ).** Put each group in the class size you found to be best for it in Problem 3. Compute the net benefit per student for each group, then the total across all 300 students.
  - (b) **Policy B (one size for everyone).** The district sets every class at 12. Compute the net benefit per student for each group, then the total across all 300 students.
  - (c) Which policy produces more? By how much? Which group accounts for most of the difference?
  - (d) Policy A is tracking. State the strongest case for it that this model supports, and then the strongest objection to it that this model cannot see.
  - (e) Suppose a student's own  $p$  falls when she is placed in a class where others disrupt. How would that change your assessment of Policy A?

**Part 3: True or False**

5. True or False, and explain in one or two sentences.
- (a) "Assigning students to classrooms by lottery solves the problem of measuring peer effects."
  - (b) "High-ability classmates raise low-ability students' scores, so a district can raise district-wide achievement by spreading its strongest students evenly across schools."
  - (c) "This school's students score well above the district average, so it is a good school."
  - (d) "Lazear's model says that smaller classes produce more learning, so classes should be made as small as the budget allows."
  - (e) "The peer effect was estimated from squadrons assigned at random, so it tells us what will happen when the academy uses it to design squadrons."