

Lecture 6 – The Marginal Principle and Class Size

The Puzzle

Project STAR found that putting K–3 students in smaller classes raised test scores. A superintendent reads this and proposes cutting every class in the district by one student. Is that a good idea? What would you need to know?

“How Many?” Decisions

Most education decisions are not yes or no. They are how much.

Students: How many hours should I study? How many credits should I take? Should I pick up one more shift?

Administrators: How many students per class? How many counselors? How many days in the school year?

The trick is to break a “how many?” decision into a series of “one more or not?” decisions.

Marginal benefit (MB): how much the benefits go up if we add one more.

Marginal cost (MC): how much the costs go up if we add one more.

This works in reverse too. Cutting one program or adding one student to a class is also a decision at the margin.

Marginal Benefits

You are studying for a midterm:

Hours	Exam score	MB (points)	MC
0	60	–	–
1	72	12	1
2	80	8	2
3	85	5	4
4	88	3	6
5	89	1	9

(MC is in the same units as points: think of it as how much you value what you give up.)

Total benefits keep rising. Is the 5th hour worth it? – *Total benefits say more is better. The margin says no.*

Why does each hour add less? – *You study the most important material first, and you get tired.* This is **diminishing marginal benefit**.

Admin examples:

Counselors. Going from 500 students per counselor to 250 lets counselors get to crises and college applications. Going from 100 to 50 adds much less.

Tutoring. A good district gives the first tutors to the students furthest behind. Each additional tutor serves students with less to gain.

Is MB always diminishing? Think about class size. Is cutting from 15 to 14 worth more or less than cutting from 25 to 24? – *It's not obvious, and the STAR data are noisy. Some benefits, like finishing a degree, only show up past a threshold.*

Marginal Costs

Back to studying. Why does MC rise? – *The first hour comes out of Netflix. The fifth comes out of sleep. You give up the least valuable things first.* This is **increasing marginal cost**.

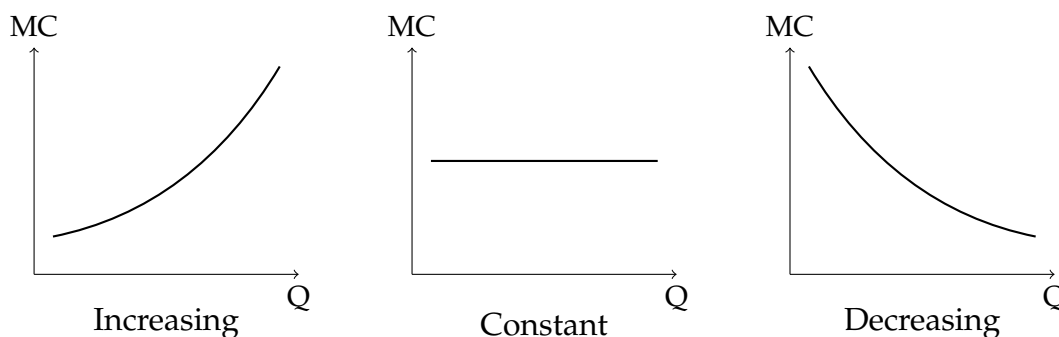
Class size. A school has 100 third graders and each teacher costs \$70,000.

Teachers	Class size	Drop in class size	Cost per 1-student drop
4	25	–	–
5	20	5.0	\$14,000
6	16.7	3.3	\$21,000
7	14.3	2.4	\$29,400
8	12.5	1.8	\$39,200

Same salary each time, but each teacher buys a smaller drop. The cost of cutting class size by one student rises as classes get smaller.

What else makes MC rise? – *You hire from deeper in the applicant pool and run out of classrooms.* California cut K–3 classes to 20 in 1996 and needed roughly 25,000 new teachers at once. Many were uncertified, and higher-income districts hired teachers away from poorer ones.

MC is not always increasing:



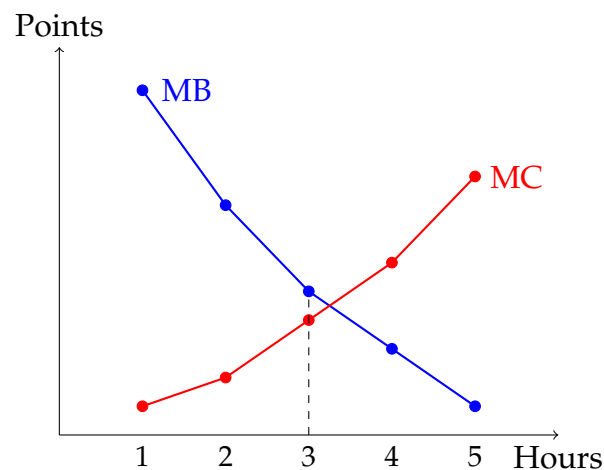
Constant: one more laptop or textbook per student. A Pell Grant for one more student. Tuition charged per credit hour.

Decreasing: one more student in an online course that already exists. A small rural district, where per-pupil costs fall as enrollment grows.

Zero: flat-rate tuition above 12 credits. What does the 5th class cost? – *No tuition, just time and effort.*

The Rule

Keep adding one more as long as $MB \geq MC$.



Study 3 hours. The 3rd hour is worth it ($5 \geq 4$). The 4th is not ($3 < 6$).

What changes the answer?

The exam is worth more of your grade. – *MB shifts up, study more.*

Your boss offers overtime. – *MC shifts up, study less.*

Teacher salaries rise. – *Cutting class size costs more, so the best class size goes up.*

Sunk costs. A district spent \$2 million on a curriculum that isn't working. Does the \$2 million matter for whether to keep it? – *No. It's spent either way. Only the costs and benefits from here forward count.*

Average vs. Marginal

STAR cut class size from about 22 to about 15 and found sizable average gains. That is an *average* effect over a 7-student cut, in a sample of schools, at a time when Tennessee did not have to hire many new teachers.

Back to the puzzle. What does STAR tell you about cutting from 22 to 21 statewide? – *MB at 22 may differ from the average over 22 to 15. MC at scale is higher: you need many new teachers and rooms at once (California).*

The real question for a superintendent is not “do smaller classes help?” It is whether the next dollar on smaller classes buys more than the next dollar on teacher pay, tutoring, or pre-K.

Splitting a Budget

A principal has \$400,000. Tutors and counselors each cost \$100,000. MB is the number of students who get back on track:

	Tutors	Counselors
1st	40	35
2nd	30	25
3rd	20	15
4th	10	5

Spend each \$100,000 where it buys the most: 1st tutor (40), 1st counselor (35), 2nd tutor (30), 2nd counselor (25). Two of each helps 130 students. All tutors helps 100.

The 1st tutor helps more students than the 1st counselor. Why not spend it all on tutors? – *By the 3rd tutor, MB has fallen below what the next counselor buys.*

When costs differ, compare **MB per dollar**. Keep moving money until the last dollar buys about the same amount everywhere.

Thresholds and Lumpy Decisions

Sometimes MB or MC comes in chunks.

Benefits. The 4th year of college pays off much more than the 3rd, because that’s when you get the degree. A student who looks only at the 3rd year may stop, when finishing is worth it.

Costs. A school’s classrooms hold 25 and it has 100 students in 4 classes. What does the 101st student cost? – *Either a crowded class or a whole new teacher and room.* The students after that are nearly free.

When decisions are lumpy, thinking one student at a time misleads. Compare the totals of the real options: 4 classes of 26 vs. 5 classes of 21 plus a teacher.

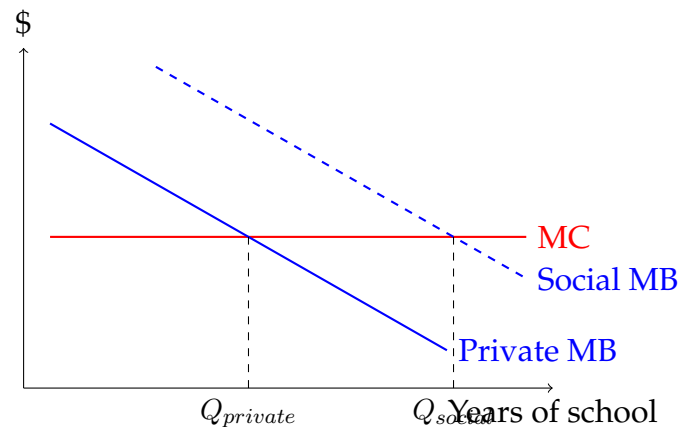
Who Pays and Who Benefits

A student decides how much school to get by comparing *her own* MB and MC.

Private MB: higher earnings, a more interesting job.

Social MB: all of that, plus more taxes paid, less crime, more voting. Other people get these benefits.

Private MC: tuition and forgone earnings. Taxpayers cover the rest of the cost.



When others benefit, students stop short of what's best for society.

If students only count their own benefits, they get less schooling than society wants. *This is one reason governments subsidize education*: a subsidy lowers the student's MC and moves them toward Q_{social} .

What does this imply for compulsory schooling, free community college, or financial aid? – *The case for each rests on how big the spillovers are.*

Beyond the Classroom

A superintendent has \$1 million. Option A cuts K–3 classes from 22 to 21. Option B hires reading specialists. What do you need to know to decide at the margin?

A district wants to add 5 days to the school year. Is the MB of day 181 the same as day 1? What is the MC, for teachers and for students?

A student on flat-rate tuition is deciding whether to drop a 5th class. What are the MB and MC?

Wrapping Up

Break “how many?” into “one more or not?”

MB is often diminishing. **MC** is often increasing, but can be constant or decreasing.

Keep going while $MB \geq MC$. Ignore sunk costs.

An average effect is not a marginal effect, especially when a policy is scaled up.

With a fixed budget, spend each dollar where MB per dollar is highest.

When benefits or costs come in chunks, compare the totals of the real options.

Students weigh private MB and MC. When others benefit, there is a case for subsidies.

Practice Problems

1. A district is deciding how many reading tutors to hire. Each tutor costs \$50,000. The district values each additional student reading at grade level at \$2,000.

Tutors	Students at grade level
0	0
1	40
2	70
3	90
4	100
5	105

- Find the MB of each tutor in students and in dollars.
 - How many tutors should the district hire?
 - A state grant cuts the cost of a tutor to \$30,000. Now how many?
 - Back at \$50,000, the district now values each student at \$3,000. Now how many?
 - Why might MB be diminishing here?
2. A school has 120 students in a grade. Each teacher costs \$60,000.
- Find class size with 4, 5, 6, 7, and 8 teachers.
 - Find the cost of each one-student drop in class size as you add the 5th, 6th, 7th, and 8th teacher. Is MC increasing, constant, or decreasing?
 - Suppose each one-student drop is worth \$20,000 to the school. How many teachers should it hire?
 - Give two reasons MC might rise even faster in practice.
3. Is the marginal cost increasing, constant, or decreasing? Explain.
- Adding one more student to an online course.
 - Giving one more student a laptop.

- (c) Adding more sections of AP Calculus when the school has one certified teacher.
 - (d) Adding one more student to a bus route.
4. A university charges \$400 per credit hour up to 12 credits and a flat \$4,800 for 12 or more.
- (a) What is the marginal tuition cost of the 9th credit hour? The 15th?
 - (b) Would you expect students to take more credits under this policy or under \$400 per credit with no cap? Why?
 - (c) Besides tuition, what are the MCs of a 15th credit hour?
5. A study finds that cutting class size from 22 to 15 raised scores by 0.21 SD, or 0.03 SD per student. A superintendent concludes that cutting her district's classes from 30 to 29 will raise scores by 0.03 SD. Give two reasons she might be wrong.
6. A district spent \$2 million on a reading curriculum. New evidence shows another curriculum works better and would cost \$500,000 to adopt. A board member says, "We can't switch, we already spent \$2 million." What costs and benefits should the board compare?
7. A principal has \$150,000. A reading specialist costs \$50,000 and a summer school section costs \$25,000. MB is the number of students who reach proficiency:

	Specialists	Summer sections
1st	30	20
2nd	20	15
3rd	10	10
4th	–	5

- (a) Find the MB per \$10,000 for each option.
 - (b) How should the principal spend the money? How many students reach proficiency?
 - (c) The 1st specialist helps more students than the 1st summer section. Why isn't that enough to decide?
8. A school has 100 students in 4 classes of 25. Five more students enroll. Opening a 5th class costs \$70,000.
- (a) If the school opens a 5th class, what is the MC of the 101st student? Of the 102nd through 105th?
 - (b) Why is comparing the MB and MC of one more student misleading here? What should the school compare instead?

9. A year of community college costs \$10,000 to provide. A student who attends gives up \$4,000 in earnings and gains \$8,000 in lifetime earnings. Society gains another \$7,000 from the student's higher taxes and lower use of public services.

- (a) If the student pays the full \$10,000, does she enroll?
- (b) Is her enrolling good for society?
- (c) The state pays \$7,000 and tuition falls to \$3,000. Does she enroll now?
- (d) How big must the spillover to society be to justify the subsidy?

Answer Key**1.**

(a)

Tutor	MB (students)	MB (\$)
1st	40	\$80,000
2nd	30	\$60,000
3rd	20	\$40,000
4th	10	\$20,000
5th	5	\$10,000

(b) 2 tutors. The 2nd is worth it ($\$60,000 \geq \$50,000$). The 3rd is not ($\$40,000 < \$50,000$).

(c) 3 tutors. The 3rd is now worth it ($\$40,000 \geq \$30,000$). The 4th is not ($\$20,000 < \$30,000$).

(d) MB is now \$120,000, \$90,000, \$60,000, \$30,000, \$15,000. Hire 3, since the 3rd is worth \$60,000 and the 4th only \$30,000.

(e) The first tutors work with the students who are closest to grade level or furthest behind and most responsive. Each new tutor reaches students who are harder to move, or who would have reached grade level anyway.

2.

(a) 30, 24, 20, 17.1, 15.

(b)

Teacher	Drop in class size	Cost per 1-student drop
5th	6.0	\$10,000
6th	4.0	\$15,000
7th	2.9	\$21,000
8th	2.1	\$28,000

MC is increasing.

(c) 6 teachers, for a class size of 20. The 6th teacher costs \$15,000 per one-student drop, which is less than \$20,000. The 7th costs \$21,000, which is more.

(d) New hires may be less experienced or uncertified. The school may need more classrooms. Higher-income districts may bid teachers away.

3.

(a) Decreasing (close to zero). The course is already built and the instructor is already paid.

- (b) Constant. Each laptop costs the same.
- (c) Increasing. The first section is cheap. More sections mean hiring or training another certified teacher, or paying overtime.
- (d) Zero until the bus is full, then a big jump for a new bus and driver.

4.

- (a) \$400 for the 9th. \$0 for the 15th.
- (b) More under the flat rate. Above 12 credits the marginal tuition cost is zero, so students keep adding credits as long as the MB beats the time and effort costs.
- (c) Study time, less time for work or family, and a risk to GPA from spreading effort across more classes.

5. Answers will vary.

The 0.03 is an average over a 7-student cut. The MB of going from 30 to 29 could be smaller or larger.

Her district may not look like the study's schools, so the effect may not generalize.

Cutting every class at once means hiring many new teachers, who may be less effective (California). This lowers MB and raises MC.

6. The \$2 million is sunk. It's spent whether they switch or not. The board should compare the \$500,000 cost of switching (plus training time) to the gains in reading from the better curriculum.

7.

(a)

	Specialists	Summer sections
1st	6	8
2nd	4	6
3rd	2	4
4th	–	2

(b) Go in order of MB per dollar: 1st summer section (8), 1st specialist (6), 2nd summer section (6). That uses \$100,000. With \$50,000 left, the 2nd specialist (20 students) beats the 3rd and 4th summer sections together (15). Two specialists and two summer sections: $30 + 20 + 20 + 15 = 85$ students. Three specialists gets 60. One specialist and four sections gets 80.

(c) The specialist costs twice as much. What matters is students per dollar, and the 1st summer section buys more.

8.

(a) \$70,000 for the 101st. Close to \$0 for each of the next four.

(b) The cost comes in one chunk. No single student is worth \$70,000, but that's the wrong question. Compare 4 classes of about 26 with 5 classes of 21 plus \$70,000.

9.

(a) No. Her MC is $\$10,000 + \$4,000 = \$14,000$, more than her MB of \$8,000.

(b) Yes. Social MB is $\$8,000 + \$7,000 = \$15,000$, more than the social MC of \$14,000.

(c) Yes. Her MC is now $\$3,000 + \$4,000 = \$7,000$, less than \$8,000.

(d) Social MB must be at least \$14,000, so the spillover must be at least \$6,000.