

## Lecture 1 – Potential Outcome Framework

### Potential Outcomes Framework

This framework underlies most modern empirical work in economics. It illustrates the fundamental problem of causal inference—we only observe one version of the world.

To illustrate: Let's say that you can't sleep and so you drink a warm glass of milk. After drinking the milk you fall asleep quickly. Did drinking warm milk cause you to fall asleep? In order to answer this we would need to see what happened if you didn't drink milk—maybe you would have fallen asleep quickly anyway. The problem is that you can't know what would have happened if you didn't drink the milk...because you didn't.

### Rubin Causal Model

This thinking is formalized in the Rubin Causal model. This model is sometimes called the **Potential Outcomes Framework**. Here is some important notation:

$Y_i$  is the outcome for person  $i$

$D_i$  is an indicator for if person  $i$  is treated. If person  $i$  is treated then  $D_i = 1$ , if they are not  $D_i = 0$

$Y_{i0}$  is the outcomes when person  $i$  is not treated and  $Y_{i1}$  be the outcome when the person  $i$  is treated.

In the previous example,  $Y_i$  is how fast person  $i$  falls asleep.  $Y_{i0}$  is how fast person  $i$  falls asleep if they don't drink milk and  $Y_{i1}$  is how fast they fall asleep if they did drink warm milk.  $D_i$  is whether person  $i$  drank milk

Given this notation we can write the following equation for  $Y_i$

$$Y_i = D_i \cdot Y_{i1} + (1 - D_i) \cdot Y_{i0}$$

The causal effect of treatment for person  $i$  is  $\alpha_i = Y_{i1} - Y_{i0}$

We only observe  $Y_i$ , so we can't calculate the causal effect of treatment for person  $i$ . This is the **fundamental problem of causal inference**.

We will never be able to recover  $\alpha_i$ , it is fundamentally unknowable. We can, however, get information that is related to  $\alpha_i$  which is the goal of this class.

Think about this in your life. How would your life be different if you decided to go to a different university? Can you know the answer to that?

$E[Y_i]$  is the expected value of  $Y_i$ . This is just the weighted average.

For example, let's say you have a sample of dogs. Let  $Y_i$  be the number of spots on the dog. There 25% of dogs have 4 spots and 75% of dogs have 8. What is  $E[Y_i]$ ? It

would be  $E[Y_i] = .25 * 4 + .75 * 8 = 7$

This is the case for discrete categories but it can be generalized to continuous measures (e.g. an infinite number of categories of dogs instead of just 2).

$E[Y_i|D = 1]$  is the conditional expectation of  $Y_i$ . In this case it is conditional on  $D$  happening. In our example, this is the expected amount of time to fall asleep for people who drink milk

You might be tempted to think that you can recover the causal effect of milk on falling asleep by simply comparing people who drink the milk to people who don't. This would be represented by:

$$E[Y_{i1}|D_i = 1] - E[Y_{i0}|D_0 = 1]$$

Why is this thinking flawed? People can decide whether they drink milk or not. Maybe people who drink milk are more likely to be hardworking dairy farmers who fall asleep quickly from long hours working on the farm regardless of what they drink before bed. If that's true, simply comparing milk drinkers to non milk drinkers would show that people who drink milk fall asleep faster. Would this be informative of the causal effect of milk?

Hence  $E[Y_{i1} - Y_{i0}]$  will not give you the causal effect of drinking milk. Instead it will give you the causal effect of drinking milk AND the average differences in sleep time among people who decide to drink milk versus people who do not. We call this second piece—the differences in the outcomes based on who chooses to be treated—**selection bias**.

### Ordinary Least Squares (OLS) Review

What do you remember about OLS? In this class the two most important things will be 1) understanding what an estimating equation is saying in words and 2) proper interpretation of coefficients.

Let our regression model be

$$Y_i = \alpha + \beta X + \varepsilon_i \quad (1)$$

OLS estimates parameters  $\hat{\alpha}, \hat{\beta}$  by minimizing the sum of squared errors. This would problem is

$$\min_{\hat{\alpha}, \hat{\beta}} \sum_{i=1}^N [(Y_i - \alpha - \beta X)^2]$$

What does the coefficient  $\beta$  tell us? What doesn't it tell us?

Why should we care about causality? The relationship between grants for college and graduation is negative—more grants, less likely to graduate. Should a policy-maker infer that grants are bad for education outcomes? Why or why not?

In the two variable case  $\hat{\beta}$  is the covariance divided by the variance ( $\frac{cov(X,Y)}{var(X)}$ ).<sup>1</sup>

Why might two things be correlated? (causal, reverse causality, omitted variables, determined jointly in a market equilibrium) (eating vegetables and height)

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<sup>1</sup>The general case is  $\hat{\beta} = (X'X)^{-1}X'Y$